\[
G = \int \eta_1(\vec{A}, \alpha_1, \ell_1, m_1, n_1)(\vec{r}_A) \eta_2(\vec{B}, \alpha_2, \ell_2, m_2, n_2)(\vec{r}_B) dV_1 dV_2
\]\[
\times \frac{1}{r_{12}} \frac{\eta_3(\vec{C}, \alpha_3, \ell_3, m_3, n_3)(\vec{r}_C) \eta_4(\vec{D}, \alpha_4, \ell_4, m_4, n_4)(\vec{r}_D)}{\ell_{12}^{\ell_1 + \ell_2} \ell_{34}^{\ell_3 + \ell_4} \ell_{12}^{\ell_1 + \ell_2} \ell_{34}^{\ell_3 + \ell_4} \ell_{12}^{\ell_1 + \ell_2} \ell_{34}^{\ell_3 + \ell_4}} dV_1 dV_2
\]
\[
= \Omega \sum_{\ell=0}^{\ell_{12}} \sum_{r=0}^{\ell_1 + \ell_2} \sum_{\ell' = 0}^{\ell_3 + \ell_4} \sum_{s'=0}^{\ell_1 + \ell_2} \sum_{i=0}^{\ell_1 + \ell_2}

B_{\ell,\ell',r,r',i}(\ell_1, \ell_2; \ell_3, \ell_4) \sum_{m_1,n_1} \sum_{m_2,n_2} \sum_{m_3,n_3} \sum_{m_4,n_4}

\times \exp \left( -\frac{\alpha_1 \alpha_2 \vec{A} \vec{B}}{\gamma_1} - \frac{\alpha_3 \alpha_4 \vec{C} \vec{D}}{\gamma_2} \right)
\]

where

\[
\nu = \ell + \ell' + m + m' + n + n' - 2(r + r' + s + s' + t + t') - (i + j + k)
\]
\[
\delta = \frac{1}{4\gamma_1} + \frac{1}{4\gamma_2}
\]

and the “Gaussian product factor” \( \Omega \), involving the original factors \( K_1 \) and \( K_2 \), is given by

\[
\Omega = \frac{2\pi^2}{\gamma_1 \gamma_2} \left( \frac{\pi}{\gamma_1 + \gamma_2} \right)^{1/2} \exp \left( -\frac{\alpha_1 \alpha_2 \vec{A} \vec{B}}{\gamma_1} - \frac{\alpha_3 \alpha_4 \vec{C} \vec{D}}{\gamma_2} \right)
\]

One thing is clear from this rather awesome formula; the electron-repulsion integral is simply a weighted sum of the integrals \( F_\nu(x) \). The coefficients which multiply the \( F_\nu(x) \) involve

1. The powers of \( x, y, \) and \( z \) in the Cartesian factors of each of the GTFs.
2. The exponents of each of the GTFs.
3. The components of the position vectors of each GTF.

The $B$ terms can be simplified by the definition of

$$\theta(\ell, \ell_1, \ell_2, a, b, r, \gamma) = f_\ell(\ell_1, \ell_2, a, b) \frac{\ell! r^{\gamma - \ell}}{r!(\ell - 2r)!}$$

Then the $B$ involving the “$x$-components” is

$$B_{\ell, \ell', r_1, r_2, i}(\ell_1, \ell_2, A_x, B_x, F_x, \gamma_1; \ell_3, \ell_4, C_x, D_x, Q_x, \gamma_2) = (-1)^{i'} \theta(\ell, \ell_1, \ell_2, P A_x, P B_x, r, \gamma_1) \theta(\ell', \ell_3, \ell_4, Q C_x, Q D_x, r', \gamma_2) \times \frac{(-1)^{i'} (2)^{2(r + r')} (\ell + \ell' - 2r - 2r')! \delta^i p_x^\ell p_x^{\ell'} - 2(r + r' + i)}{(4 \delta)^{\ell + \ell' i}[\ell + \ell' - 2(r + r' + i)]!}$$

With completely analogous expressions for the other two $B$ factors.